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Solution of a Problem of Generalized Thermoelasticity of an Annular Cylinder with Variable Material Properties by Finite Difference Method

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Abstract: The present work deals with a new problem of generalized thermoelasticity with one relaxation time for an infinitely long and isotropic annular cylinder of temperature dependent physical properties. The inner and outer curved surfaces of the cylinder are subjected to both the mechanical and thermal boundary conditions. A finite difference model is developed to derive the solution of the problem in which the governing equations are coupled non linear partial differential equations. The transient solution at any time can be evaluated directly from the model. In order to demonstrate the efficiency of the present model we consider a suitable material and obtain the numerical solution of displacement, temperature, and stresses inside the annulus for both the temperature-dependent and temperature-independent material properties of the medium. The results are analyzed with the help of different graphical plots.

Keywords: generalized thermoelasticity, thermoelasticity with one relaxation parameter, annular cylinder, finite difference method.

I. INTRODUCTION

During last few decades a considerable development of the subject, thermoelasticity is motivated by various fields of engineering science. The theory of uncoupled thermoelasticity suffers from physical drawbacks that mechanical state of elastic body has no effect on the temperature and vise versa and also the thermal signal propagates with infinite speed. Based on thermodynamics principles of irreversible processes, Biot [1] gave a satisfactory derivation of the equation of thermal conductivity by including the coupling between thermal and strain fields and gave rise to the coupled theory of thermoelasticity. This theory removes the first drawback but shares the second defect of the uncoupled theory due to the presence of parabolic type heat conduction equation. Later on, eforts to remove this drawback led to the generalized theories of thermoelasticity in which the heat conduction equation is of hyperbolic type. Lord and Shulman [2] developed a theory of generalized thermoelasticity with one relaxation time, also known as extended thermoelasticity theory, by introducing the time derivative of the heat flux vector and a new constant acting as a relaxation time in classical Fourier's law. The heat conduction equation of this theory is of wave type, ensuring finite speed of propagation of both the thermal wave and elastic wave and thus removes the second drawback of the uncoupled theory. This theory was extended by Dhaliwal and Sherief [3] to include the effect of anisotropic behaviour and the presence of heat source. Another well established generalized thermoelasticity theory is due to Green and Lindsay [4]. This theory is also referred as the temperature-rate dependent theory of thermoelasticity and takes into account two relaxation times. Later on, another three new formulations of thermoelasticity based on entropy balance inequality have been proposed by Green and Naghdi [5-7]. Several experimental studies [8-10] also indicate the actual occurrence of wave type heat transport. Subsequently, several investigations [11-16] are carried out on the basis of different generalized theories of thermoelasticity.

In most of the problems the material properties of the medium are taken to be constant. However, it is well known that the physical properties of engineering materials very considerably with temperature. At high temperature the material properties like modulus of elasticity, Poisson's ratio, thermal conductivity etc. no longer remain constants and the temperature dependency of material properties affect the thermo-mechanical behavior of the medium [17]. Hence to obtain more dependable solution of the problem of thermoelasticity, temperature dependency of material properties should be taken into consideration. Recently, investigations pertaining to thermoelastic deformation of several basic structures like disk, cylinders, tubes etc. are being carried out [18-24] by taking into account the temperature dependent properties of the medium. Two different problems in generalized thermoelasticity for an infinitely long annular cylinder with constant material properties were studied by Sherief and Anwar [25, 26].

The aim of the present paper is to investigate a problem of an infinitely long annular cylinder, whose material properties like modulus of elasticity and thermal conductivity vary with temperature, in the context of generalized thermoelasticity theory with one relaxation parameter. The governing equations in this case are coupled nonlinear partial differential equations because of varying material parameters. Using the finite difference method the governing equations are transformed into a system of coupled difference equations and the numerical solution for the field variables inside the annulus for copper material are obtained directly in space-time domain. Results are displayed graphically and compared with the results obtained for temperature-independent material properties.

Nomenclature

λ, μ	Lame's elastic constants,
T	absolute temperature,
T_0	reference temperature,
u_i	components of displacement vector,
e_{ij}	components of strain tensor,
σ_{ij}	components of stress tensor,
K	thermal conductivity,
ρ	density,
κ	thermal diffusivity,
$\Delta = e_{ii}$	dilatation,
τ_0	thermal relaxation time parameter,
δ_{ij}	Kronecker's delta,

$\gamma = (3\lambda + 2\mu)\alpha_t$, where α_t is the coefficient of linear thermal expansion.

II. GOVERNING EQUATIONS

We consider an isotropic elastic medium with temperature dependent material properties. The constitutive relations and basic equations based on the generalized thermoelastic model with one relaxation time due to Lord and Shulman [2] can be written as follows:

Stress-strain-temperature relations:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda\Delta - \gamma\overline{T})\delta_{ij}, \tag{1}$$

where

$$\overline{T} = T - T_0.$$

Strain-displacement relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2}$$

Equation of motion in absence of body forces:

$$\sigma_{ij,j} = \rho\ddot{u}_i. \tag{3}$$

Heat conduction equation in absence of heat sources:

$$\left(K\overline{T}_{,i}\right)_{,i} = \frac{K}{\kappa}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\overline{T} + \gamma T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\Delta. \tag{4}$$

We assume that

$$\lambda = \lambda_0 f(T),\ \mu = \mu_0 f(T),\ K = K_0 f(T),\ \gamma = \gamma_0 f(T),$$

where λ_0, μ_0, K_0 and γ_0 are considered to be constants, and $f(T)$ be a given non dimensional function of temperature. In case of temperature-independent material properties $f(T) = 1$ and $\lambda = \lambda_0, \mu = \mu_0, K = K_0$, $\gamma = \gamma_0$ (Noda [17], Youssef and Abbas [23]).

Therefore from equations (1)-(4) we get

$$\sigma_{ij} = \left[2\mu_0 e_{ij} + \left(\lambda_0\Delta - \gamma_0\overline{T}\right)\delta_{ij}\right]f(T), \tag{5}$$

$$\rho\ddot{u}_i = \left[2\mu_0 e_{ij} + \left(\lambda_0\Delta - \gamma_0\overline{T}\right)\delta_{ij}\right]_{,j} f(T) + \left(f(T)\right)_{,j}\left[2\mu_0 e_{ij} + \left(\lambda_0\Delta - \gamma_0\overline{T}\right)\delta_{ij}\right], \tag{6}$$

$$\left[K_0 f(T)\overline{T}_{,i}\right]_{,i} = \frac{K_0 f(T)}{\kappa}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\overline{T} + \gamma_0 f(T) T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\Delta, \tag{7}$$

III. PROBLEM FORMULATION

We consider an infinitely long annular cylinder consisting of isotropic elastic material with temperature dependent material properties. We introduce the cylindrical polar co-ordinates (r,φ,z) with the origin at the centre of the cylindrical polar co-ordinate system.

Axi-symmetric plane strain problem for which the physical quantities are assumed to be the functions of radial coordinate r and time t is considered for our analysis.

For the simplicity of the problem we approximate the function $f(T)$ as $f(T)=1-\alpha T$, where α is a material parameter of the dimension K^{-1} (Noda [17]).

Therefore from (5) we get the non-zero stress components as

$$\sigma_{rr}=\left[(\lambda_0+2\mu_0)\frac{\partial u}{\partial r}+\lambda_0\frac{u}{r}-\gamma_0\bar{T}\right](1-\alpha T), \tag{8}$$

$$\sigma_{\varphi\varphi}=\left[(\lambda_0+2\mu_0)\frac{u}{r}+\lambda_0\frac{\partial u}{\partial r}-\gamma_0\bar{T}\right](1-\alpha T). \tag{9}$$

Equation (3) then yields the equation of motion in the cylindrical form as

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\left(\sigma_{rr}-\sigma_{\varphi\varphi}\right)=\rho\frac{\partial^2 u}{\partial t^2}. \tag{10}$$

From (8), (9) and (10) we obtain:

$$\begin{gathered}(\lambda_0+2\mu_0)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right](1-\alpha T)+\\ -\left[\alpha\left\{(\lambda_0+2\mu_0)\frac{\partial u}{\partial r}+\lambda_0\frac{u}{r}-\gamma_0\bar{T}\right\}+\gamma_0(1-\alpha T)\right]\frac{\partial\bar{T}}{\partial r}=\\ =\rho\frac{\partial^2 u}{\partial t^2}.\end{gathered} \tag{11}$$

Equation (7) yields

$$\begin{gathered}\left(\frac{\partial^2\bar{T}}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{T}}{\partial r}\right)-\frac{\alpha}{1-\alpha T}\left(\frac{\partial\bar{T}}{\partial r}\right)^2=\\ =\frac{1}{\kappa}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\bar{T}+\frac{\gamma_0 T_0}{K_0}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right).\end{gathered} \tag{12}$$

For our convenience, we now introduce the following non dimensional variables and notations:

$$r'=\frac{c_0 r}{\kappa},\ u'=\frac{c_0 u}{\kappa},\ t'=\frac{c_0{}^2 t}{\kappa},$$

$$\tau_0'=\frac{c_0{}^2\tau_0}{\kappa},\ \theta'=\frac{\bar{T}}{T_0},\ \sigma_{ij}'=\frac{\sigma_{ij}}{(\lambda_0+2\mu_0)},\ \lambda_1=\frac{\lambda_0}{\lambda_0+2\mu_0}$$

$$a_1=\frac{\gamma_0 T_0}{(\lambda_0+2\mu_0)},\quad a_2=\frac{\gamma_0\kappa}{K_0},\quad \beta=\alpha T_0.$$

Therefore, the dimensionless forms of equations (8), (9), (11) and (12) are obtained as follows (after dropping the primes for convenience):

$$\begin{gathered}\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]\left[1-\beta(\theta+1)\right]+\\ -\left[a_1\left[1-\beta(2\theta+1)\right]+\beta\left(\frac{\partial u}{\partial r}+\lambda_1\frac{u}{r}\right)\right]\frac{\partial\theta}{\partial r}=\frac{\partial^2 u}{\partial t^2},\end{gathered} \tag{13}$$

$$\begin{gathered}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\theta=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\theta+\\ +a_2\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)+\frac{\beta}{1-\beta(\theta+1)}\left(\frac{\partial\theta}{\partial r}\right)^2,\end{gathered} \tag{14}$$

$$\begin{gathered}\sigma_{rr}=\left[1-\beta(\theta+1)\right]\times\\ \times\left[\frac{\partial u}{\partial r}+\lambda_1\frac{u}{r}-a_1\theta\right],\end{gathered} \tag{15}$$

$$\begin{gathered}\sigma_{\varphi\varphi}=\left[1-\beta(\theta+1)\right]\times\\ \times\left[\lambda_1\frac{\partial u}{\partial r}+\frac{u}{r}-a_1\theta\right].\end{gathered} \tag{16}$$

We assume that initially the annulus has no deformation and have the reference temperature T_0 and also the zero rate of change of temperature.

Therefore initial conditions may be expressed as

$$u(r,0)=\frac{\partial u(r,0)}{\partial t}=0,\quad \theta(r,0)=\frac{\partial\theta(r,0)}{\partial t}=0 \tag{17}$$

$$a\le r\le b,$$

where a and b are the dimensionless inner and outer radii of the cylinder.

It is assumed that both the inner and outer curved surfaces of the annulus are stress free and the inner surface is subjected to a temperature which is decaying with time, whereas the outer surface is maintained at the reference temperature. The boundary conditions are therefore taken to be as follows:

$$\sigma_{rr}=0,\ \theta=e^{-\omega t}\quad \text{at}\quad r=a,\quad t>0, \tag{18}$$

$$\sigma_{rr}=0,\ \theta=0\quad \text{at}\quad r=b,\quad t>0, \tag{19}$$

where ω is the decaying exponent.

IV. SOLUTION OF THE PROBLEM

The governing equations obtained in the last section are non linear partial differential equations. For the solution of the problem we use the finite difference method. The solution domain $a \le r \le b,\ 0 \le t \le \tau$ is replaced by a grid described by the set of node points (r_m, t_n), in which $r_m = a + mh,\ m = 0,1,...,N$ and $t_n = nk;\ n = 0,1,...,P$. Therefore $h = (b-a)/N$ is taken as mess width and $k = \tau/P$ is assumed to be the time step. Also we assume that τ is the final value of time. In the following equations we use the notation u_m^n in place of $u(r_m, t_n)$, m = 0, 1,…, N and n = 0, 1,…, P. The finite difference approximations for the partial differential coefficients with respect to the independent variables r and t are obtained as follows:

$$\frac{\partial y}{\partial r} = \frac{y_{m+1}^n - y_{m-1}^n}{2h} + o\left(h^2\right), \quad \frac{\partial^2 y}{\partial r^2} = \frac{y_{m+1}^n - 2y_m^n + y_{m-1}^n}{h^2} + o(h^2), \quad \frac{\partial y}{\partial t} = \frac{y_m^{n+1} - y_m^{n-1}}{2k} + o\left(k^2\right) \tag{20}$$

The equations (13) and (14) are then replaced by the explicit finite difference equations as

$$u_m^{n+1} = 2u_m^n - u_m^{n-1} + \upsilon \left[\begin{array}{l} \left[1-\beta\left(\theta_m^n+1\right)\right]\left\{\left(u_{m+1}^n - 2u_m^n + u_{m-1}^n\right) + \dfrac{h}{2r_m}\left(u_{m+1}^n - u_{m-1}^n\right) - \dfrac{h^2}{r_m^{\ 2}} u_m^n\right\} \\ -\dfrac{h}{2}\left\{a_1\left[1-\beta(2\theta_m^n+1)\right] + \beta\left(\dfrac{u_{m+1}^n - u_{m-1}^n}{2h} + \lambda_1 \dfrac{u_m^n}{r_m}\right)\right\}\left(\theta_{m+1}^n - \theta_{m-1}^n\right) \end{array} \right], \tag{21}$$

$$\theta_m^{n+1} = \frac{1}{(k+2\tau_0)} \left[\begin{array}{l} 4\tau_0\theta_m^n + (k-2\tau_0)\theta_m^{n-1} + 2v\left\{\left(\theta_{m+1}^n - 2\theta_m^n + \theta_{m-1}^n\right) + \dfrac{h}{2r_m}\left(\theta_{m+1}^n - \theta_{m-1}^n\right)\right\} \\ -\dfrac{a_2 k}{2h}\left\{\left(u_{m+1}^{n+1} - u_{m+1}^{n-1} - u_{m-1}^{n+1} + u_{m-1}^{n-1}\right) + \dfrac{2h}{r_m}\left(u_m^{n+1} - u_m^{n-1}\right)\right\} \\ -\dfrac{a_2\tau_0}{h}\left\{\left(u_{m+1}^{n+1} - 2u_{m+1}^n + u_{m+1}^{n-1} - u_{m-1}^{n+1} + 2u_{m-1}^n - u_{m-1}^{n-1}\right) + \dfrac{2h}{r_m}\left(u_m^{n+1} - 2u_m^n + u_m^{n-1}\right)\right\} \\ -\dfrac{\beta v}{2\left[1-\beta\left(\theta_m^n+1\right)\right]}\left(\theta_{m+1}^n - \theta_{m-1}^n\right)^2 \end{array} \right], \tag{22}$$

where we have used the notation $\upsilon = k^2/h^2$.

Equations (15) and (16) then reduce to

$$[\sigma_{rr}]_m^n = \left[1-\beta\left(\theta_m^n+1\right)\right]\left[\frac{u_{m+1}^n - u_{m-1}^n}{2h} + \lambda_1\frac{u_m^n}{r_m} - a_1\theta_m^n\right], \tag{23}$$

$$[\sigma_{\theta\theta}]_m^n = \left[1-\beta\left(\theta_m^n+1\right)\right]\left[\frac{u_m^n}{r_m} + \lambda_1\frac{u_{m+1}^n - u_{m-1}^n}{2h} - a_1\theta_m^n\right]. \tag{24}$$

From the initial condition (17) and using equation (20) we get

$$\frac{\partial u_m^0}{\partial t} = \frac{u_m^1 - u_m^{-1}}{2k} = 0, \qquad \frac{\partial \theta_m^0}{\partial t} = \frac{\theta_m^1 - \theta_m^{-1}}{2k} = 0 \tag{25}$$

Now using equation (25) we can eliminate u_m^{-1} and θ_m^{-1} from equations (21) and (22) and get the equations satisfied by u_m^n and θ_m^n for the first level of t (n = 0) as

$$u_m^1 = u_m^0 + \frac{\upsilon}{2} \left[\begin{array}{l} \left[1-\beta\left(\theta_m^0+1\right)\right]\left\{\left(u_{m+1}^0 - 2u_m^0 + u_{m-1}^0\right) + \dfrac{h}{2r_0}\left(u_{m+1}^0 - u_{m-1}^0\right) - \dfrac{h^2}{r_0^{\ 2}} u_m^0\right\} \\ -\dfrac{h}{2}\left\{a_1\left[1-\beta(2\theta_m^0+1)\right] + \beta\left(\dfrac{u_{m+1}^0 - u_{m-1}^0}{2h} + \lambda_1\dfrac{u_m^0}{r_m^0}\right)\right\}\left(\theta_{m+1}^0 - \theta_{m-1}^0\right) \end{array} \right], \tag{26}$$

$$\theta_m^1=\theta_m^0+\left[\begin{array}{l}\frac{\upsilon}{2\tau_0}\left\{\left(\theta_{m+1}^0-2\theta_m^0+\theta_{m-1}^0\right)+\frac{h}{2r_0}\left(\theta_{m+1}^0-\theta_{m-1}^0\right)\right\}-\\-\frac{a_2}{4h}\left\{\left(2u_{m+1}^1-2u_{m+1}^0-2u_{m-1}^1+2u_{m-1}^0\right)+\frac{4h}{r_0}\left(u_m^1-u_m^0\right)\right\}\\-\frac{\beta\upsilon}{8\tau_0\left[1-\beta\left(\theta_m^0+1\right)\right]}\left(\theta_{m+1}^0-\theta_{m-1}^0\right)^2\end{array}\right] \tag{27}$$

Using the boundary condition (18) and equation (23) we get for the line $r=a$ as

$$\frac{u_1^n-u_{-1}^n}{2h}+\lambda_1\frac{u_0^n}{r_0}-\theta_0^n=0 \text{ and } \theta_0^n=e^{-\omega t_n}. \tag{28}$$

Now substituting the expression for u_{-1}^n from equation (28) into equation (21) we get the equation satisfied by u_m^n for $r=a$ (i.e. for the level $m=0$) as

$$u_0^{n+1}=2u_0^n-u_0^{n-1}+$$
$$+\upsilon\left[\begin{array}{l}\left[1-\beta\left(\theta_0^n+1\right)\right]\left\{\begin{array}{l}2\left(u_1^n-u_0^n+h\lambda_1\frac{u_0^n}{r_0}-a_1h\theta_0^n\right)\\+\frac{h^2}{r_0}\left(a_1\theta_0^n-\lambda_1\frac{u_0^n}{r_0}\right)-\frac{h^2}{r_0^2}u_0^n\end{array}\right\}\\-\frac{h}{2}\left\{a_1\left[1-\beta\left(\theta_0^n+1\right)\right]\right\}\left(-3\theta_0^n+4\theta_1^n-\theta_2^n\right)\end{array}\right]. \tag{29}$$

Using equation (19) we get for the line $r=b$ as

$$\frac{u_{N+1}^n-u_{N-1}^n}{2h}+\lambda_1\frac{u_N^n}{r_N}-\theta_N^n=0,\qquad \theta_N^n=0 \tag{30}$$

Therefore substituting u_{N+1}^n from equation (30) into equation (21) we obtain the equation for the level $m=N$ as

$$u_N^{n+1}=2u_N^n-u_N^{n-1}+\upsilon\left[\begin{array}{l}\left[1-\beta\left(\theta_N^n+1\right)\right]\left\{\begin{array}{l}2\left(u_{N-1}^n-u_N^n-h\lambda_1\frac{u_N^n}{r_N}+a_1h\theta_N^n\right)\\+\frac{h^2}{r_N}\left(a_1\theta_N^n-\lambda_1\frac{u_N^n}{r_N}\right)-\frac{h^2}{r_N^2}u_N^n\end{array}\right\}\\-\frac{h}{2}\left\{a_1\left[1-\beta\left(\theta_N^n+1\right)\right]\right\}\left(3\theta_N^n-4\theta_{N-1}^n+\theta_{N-2}^n\right)\end{array}\right] \tag{31}$$

The equations (21)-(31) therefore constitute the model of finite difference scheme for the present problem to determine the values of the physical field variables u,θ,σ_{rr} and $\sigma_{\varphi\varphi}$ at different points of the solution domain $a\le r\le b,\ 0\le t\le\tau$.

Now by Taylor series expansion and using equation (13) we obtain the truncation error (TE) of the finite difference equation (21) as

$$k^{-1}\left(TE\right)_m^n=\frac{k^3}{12}\frac{\partial^4u_m^n}{\partial t^4}+\frac{k^5}{360}\frac{\partial^6u_m^n}{\partial t^6}$$
$$-kh^2\left[\begin{array}{l}\left\{[1-\beta(\theta_m^n+1)]\left(\frac{1}{12}\frac{\partial^4u_m^n}{\partial r^4}+\frac{h^2}{360}\frac{\partial^6u_m^n}{\partial r^6}+\frac{1}{r_m}\left(\frac{1}{6}\frac{\partial^3u_m^n}{\partial r^3}+\frac{h^2}{120}\frac{\partial^5u_m^n}{\partial r^5}\right)\right)\right\}\\-h^3\left\{\beta\left(\frac{1}{6}\frac{\partial^3u_m^n}{\partial r^3}+\frac{h^2}{120}\frac{\partial^5u_m^n}{\partial r^5}\right)\right\}\left(\frac{1}{6}\frac{\partial^3\theta_m^n}{\partial r^3}+\frac{h^2}{120}\frac{\partial^5\theta_m^n}{\partial r^5}\right)\end{array}\right] \tag{32}$$

From (32) it is evident that $TE \to 0$ as $h \to 0$, $k \to 0$. Similarly, the truncation error of the finite difference equation (22) can also be shown to tend to zero as $h \to 0$ and $k \to 0$. Therefore the finite difference equations of our system and the differential equations for the present problem become equivalent, so that our finite difference scheme is consistent [27]. The truncation error is of second order accurate in space and third order accurate in time i.e. of the order $o(k^3 + kh^2)$.

V. NUMERICAL RESULTS AND DISCUSSION

In order to observe the validity and efficiency of our system of difference equations and also to get the nature of variations of different field variables like displacement, temperature and stresses inside the medium we have done numerical computations with the help of computer programming. For this purpose we consider copper material, the physical data for which is taken as follows: $\tau_0 = 0.02$, $T_0 = 819$ K, $\lambda_0 = 7.76 \times 10^{10}$ Nm^{-2}, $\mu_0 = 3.86 \times 10^{10}$ Nm^{-2}, $\rho = 8\,954$ Kgm^{-3}, $\alpha_t = 1.78 \times 10^{-5}$ K^{-1}, $\kappa = 0.000113$ m^{2}s^{-1}, $\omega = 0.1$.

The inner radius and the outer radius of cylinder are taken as 1.0 and 5.0 respectively and we assume $\tau = 1.0$ and $\upsilon = 0.0156$.

The computations are carried out for both the cases: variable material properties ($\alpha \neq 0$) and temperature independent (i.e. constant) material properties ($\alpha = 0$). We first compute the values of u and θ simultaneously for different values of our specified domain from the coupled equations (21), (22) and (26)-(31). Then the values of stresses are computed from equations (23) and (24). In order to observe the effect of the parameter α on the values of different field variables we carry out our computation for three different non zero values of the parameter α (0.00051, 0.00025, 0.00012). The results are depicted in different figures in order to show the variation of different fields with respect to the radial coordinate (Figures 1(a), 2(a), 3(a), 4(a)) as well as with respect to time (Figures 1(b), 2(b), 3(b), 4(b)).

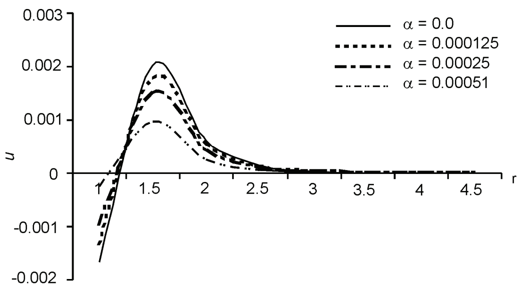

Fig. 1(a). Variation of displacement at $t = 0.4$

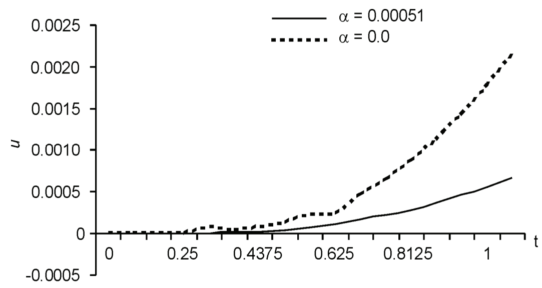

Fig. 1(b). Variation of displacement at $r = 2.5$

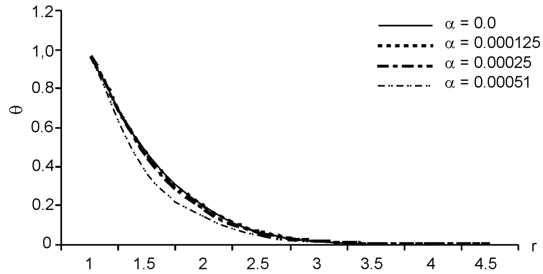

Fig. 2(a). Variation of temperature at $t = 0.4$

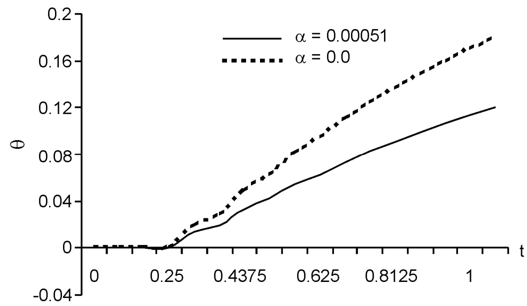

Fig. 2(b). Variation of temperature at $r = 2.5$

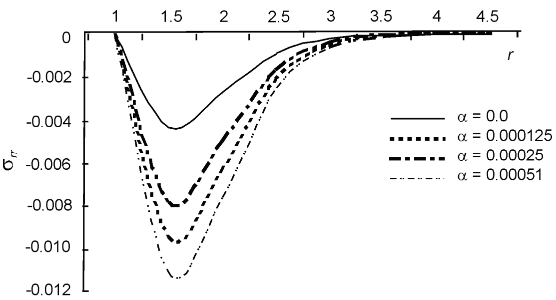

Fig. 3(a). Variation of radial stress at $t = 0.4$

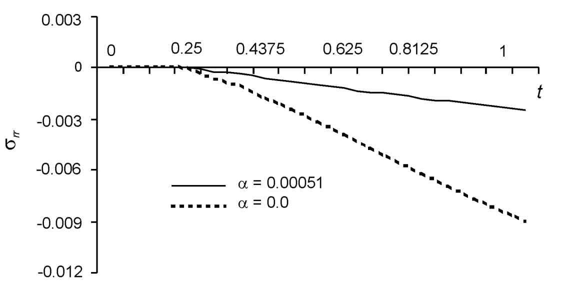

Fig. 3(b). Variation of radial stress at $r = 2.5$

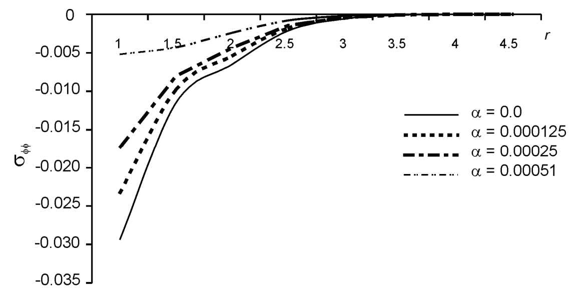

Fig. 4(a). Variation of circumferential stress at $t = 0.4$

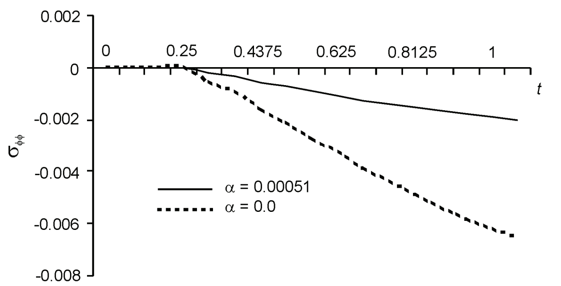

Fig. 4(b). Variation of circumferential stress at $r = 2.5$

The nature of variations of various fields observed in different figures indicates that our system of difference equations (21)-(31) efficiently compute the numerical solutions of the problem and the solutions obtained are in complete agreement with the theoretical boundary conditions of the problem. The following important facts are also evident from the graphical results:

1. Although the nature of variation of each field is similar in trend for both the cases but the fields show significantly different values in two cases.
2. At any time the field variables show numerically higher values in the case of temperature independent material properties.
3. In case of variable material parameters the absolute values of each field increases with the decrease of the parameter α and as $\alpha \to 0$ the curve of the distribution of each field for the temperature dependent case approaches to the curve of the distribution of the field for the case of temperature independent material parameters.
4. The difference between the values of physical quantities for temperature dependent material parameters and the values for temperature independent material parameters increases with time.
5. Stresses are compressive in nature for both the cases.
6. Circumferential stress and temperature shows maximum value at inner surface of the annulus.
7. The disagreement between two cases is more prominent for stress and displacement field as compared to the temperature field.

VI. CONCLUSIONS

A general finite difference model is developed to analyze a generalized coupled thermoelastic problem of an infinitely long annular cylinder with variable material properties. This kind of coupled problem, involving non-linear partial differential equations and is difficult to approach by analytical techniques, can be accurately and efficiently approached by this model. The effects of temperature dependency of the material properties on different field variables of the medium, which may be significant in some practical applications, can easily be taken under consideration and accurately assessed.

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