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Reliability Properties of Seven Parameters Burr XII Distribution

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Abstract: This paper investigates reliability properties of a flexible extended (seven parameters) Burr XII family of distributions. Moreover, closed forms for *n*-th moments are derived.

Key words: Burr XII distribution, reliability function, moments and quantiles

EB – Extended Burr
RF – Reliability Function
RV – Random Variable
Pdf – probability density function
Cdf – cumulative distribution function
FR – Failure Rate

I. INTRODUCTION

A six parameters generalized Burr XII distribution is introduced in Olapade [9]. The Burr XII distribution is an important distribution because it has many other distributions like Pareto II or Lomax distribution (see Arnold [2], Balakrishnan and Ahsanullah [5]), log-logistic distribution (Balakrishnan, Malik and Puthenpura [4]), compound-Weibull or Weibull-Gamma and Weibull Exponential (Tadikamalla [11]) as particular cases of this distribution. So generalizing Burr XII distribution is generalizing all other distributions which come under special cases of Burr XII distribution.

Adding parameters to a well established family of distributions is a time honoured device for obtaining more flexible new families of distributions. An ingeneous general method of adding a parameter to a family of distributions is introduced in [8]. Their method is as follows: if we have a family of distributions with RF $\bar{F}_0(x)$, then a new family of distributions can be constructed basing on the relation:

$$\bar{F}(x) = \frac{\gamma \bar{F}_0(x)}{1-\bar{\gamma}\bar{F}_0(x)}, \quad -\infty < x < \infty, \ \ \gamma > 0, \ \ \bar{\gamma} = 1-\gamma. \tag{1}$$

They have discovered that the method has a stability property, namely, if the method is applied twice, nothing new will be obtained the second time around. These authors have shown that the newly generated family of distributions with the RF (1) is geometric extreme stable, namely, if X_1, X_2, ... is a sequence of independent and identically distributed RVs with RF (1), independent of a RV N possessing a geometric distribution with the probability mass function:

$$P(N=n) = p(1-p)^{n-1}, \quad n = 1,2,\ldots, \ \ 0 < p < 1, \tag{2}$$

then the RVs

$$U = \min(X_1, X_2, \ldots, X_N), \quad V = \max(X_1, X_2, \ldots, X_N), \tag{3}$$

also belong to the family of distributions with the RF (1).

In this paper, we introduce a new variant of Marshall-Olkin extended family of distributions by selecting in (1) the RF of generalized Burr XII distribution:

$$\bar{F}_0(x)=\lambda^m\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^{-m}, \qquad (4)$$

$$x>0,\ \lambda, k, m, p, \mu, \sigma>0.$$

Substituting (4) in (1), we obtain

$$\bar{F}(x)=\frac{\gamma\lambda^m\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^{-m}}{1-\bar{\gamma}\lambda^m\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^{-m}}=$$

$$=\frac{\gamma\lambda^m}{\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m-\bar{\gamma}\lambda^m}, \quad x>0. \qquad (5)$$

We shall write $X\sim EB(\mu, \sigma, \gamma, \lambda, k, m, p)$ to denote an absolutely continuous RV X possessing the Marshall-Olkin extended (seven parameters) Burr XII distribution with parameters $\mu, \sigma, \gamma, \lambda, k, m, p$ and RF given by (5).

The calculations in this paper need the following important lemma.

Lemma 1.

(Gradshteyn and Ryzhik [7], Eq. (3.241.4)).

The following identity is held

$$\int_0^\infty \frac{x^{\mu-1}dx}{\left(p+qx^v\right)^{n+1}}=\frac{1}{vp^{n+1}}\left(\frac{p}{q}\right)^{\frac{\mu}{v}}B\left(\frac{\mu}{v},\ n+1-\frac{\mu}{v}\right), \qquad (6)$$

where $(0<\mu/v<n+1)$ and $B(,)$ is the beta function.

The aim of this paper is to reveal some reliability properties of EB distribution as in [6]. These properties include: (i) expressing this proposed distribution as a compounding process with exponential mixing model, (ii) shapes of the probability density function, (iii) moments and quantiles, (iv) shapes of the failure rate function, (v) stochastic ordering, and (vi) limiting distributions of extreme order statistics.

II. COMPOUNDING

Let $\bar{F}(x\mid\theta), -\infty<x<\infty,\ -\infty<\theta<\infty$, be the conditional RF of a continuous RV X given a continuous RV Θ. Let Θ follow a distribution with the pdf $m(\theta)$. A distribution with RF

$$\bar{F}(x)=\int_{-\infty}^{+\infty}\bar{F}(x\mid\theta)m(\theta)\,d\theta, \quad -\infty<x<\infty, \qquad (7)$$

is called a compound distribution with mixing density $m(\theta)$. Compound distribution provides a tool for obtaining new parametric families of distributions in terms of existing ones. They represent heterogeneous models where populations items involve different risks.

The following theorem shows that the EB distribution can be expressed by the compounding argument.

Theorem 1

Let the conditional RF of a continuous RV X given $\Theta=\theta$ be expressed as:

$$\bar{F}(x|\theta)=\exp\left(-\left(\lambda^{-m}\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m-1\right)\theta\right), \qquad (8)$$

$$x, \theta>0,\ \lambda, k, m, p, \mu, \sigma>0.$$

Let Θ follow an exponential distribution with pdf:

$$m(\theta)=\gamma e^{-\gamma\theta},\ \theta>0,\ \gamma>0. \qquad (9)$$

Then the compound distribution of X is the $EB(\mu, \sigma, \gamma, \lambda, k, m, p)$ distribution.

Proof

For all $x>0, \mu, \sigma, \gamma, \lambda, k, m, p>0,$ the unconditional RF of X is given by

$$\bar{F}(x)=\int_0^\infty \bar{F}(x\mid\theta)m(\theta)d\theta=$$

$$=\gamma\int_0^\infty \exp\left(-\left(\lambda^{-m}\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m-\bar{\gamma}\right)\theta\right)d\theta= \qquad (10)$$

$$=\frac{\gamma\lambda^m}{\left(\lambda+k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m-\bar{\gamma}\lambda^m},$$

which is indeed the RF (5) of $EB(\mu, \sigma, \gamma, \lambda, k, m, p)$ distribution.

III. THE PROBABILITY DENSITY FUNCTION

The pdf of the $EB(\mu, \sigma, \gamma, \lambda, k, m, p)$ distribution with RF (5), is given by

$$f(x)=-\frac{d\bar{F}(x)}{dx}=$$

$$= \frac{k\, m\, p\, \gamma \lambda^m \left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^{m-1} \left(\frac{x-\mu}{\sigma}\right)^{p-1}}{\left(\left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m - \bar{\gamma}\lambda^m\right)^2 \sigma}. \quad (11)$$

The following theorem gives simple conditions under which the pdf (11) is decreasing or unimodal.

Theorem 2

Let $X \sim EB(\mu, \sigma, \gamma, \lambda, k, m, p)$, then X has decreasing (unimodal) pdf provided

$$k\left((mp-1)(\gamma-1)\lambda^m - (mp+1)\left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m\right)\left(-\frac{\mu}{\sigma}\right)^p + $$
$$+ (p-1)\lambda\left((\gamma-1)\lambda^m + \left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m\right) \le 0 (> 0).$$

Proof

The first derivative of $f(x)$ is given by

$$f'(x) = \frac{k\, m\, p\, \gamma \lambda^m \left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^{m-2} \left(\frac{x-\mu}{\sigma}\right)^{p} \psi(x)}{(x-\mu)^2\left(\left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m - \bar{\gamma}\lambda^m\right)^3}, \quad (12)$$

$$x > 0,$$

where

$$\psi(x) = k\left((mp-1)(\gamma-1)\lambda^m - (mp+1)\left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^m\right) \times$$
$$\times \left(\frac{x-\mu}{\sigma}\right)^p + (p-1)\lambda\left((\gamma-1)\lambda^m + \left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^m\right).$$

The function $\psi(x)$ has no (one) zero on $(0, \infty)$ provided

$$\psi(0) = k\left((mp-1)(\gamma-1)\lambda^m - (mp+1)\left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m\right) \times$$
$$\times \left(-\frac{\mu}{\sigma}\right)^p + (p-1)\lambda\left((\gamma-1)\lambda^m + \left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m\right) \le 0 (> 0).$$

That is, $f(x)$ has no (one) critical point provided $\psi(0) \le 0 (> 0)$. Since $f(x)$ is non-negative and

$$f(0) = \frac{k\, m\, p\, \gamma \lambda^m \left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^{m-1} \left(-\frac{\mu}{\sigma}\right)^{p-1}}{\left(\left(\lambda + k\left(-\frac{\mu}{\sigma}\right)^p\right)^m - \bar{\gamma}\lambda^m\right)^2 \sigma},$$

$f(\infty) = 0$ then $f(x)$ is decreasing (unimodal) provided $\psi(0) \le 0 (> 0)$.

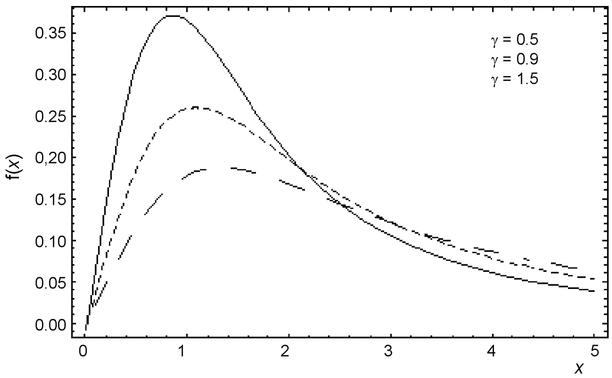

Fig. 1. Plot of pdf for different values of γ for $\mu = 0.02$, $\sigma = 1$, $\lambda = m = 0.5$, $k = 0.2$ and $p = 2$

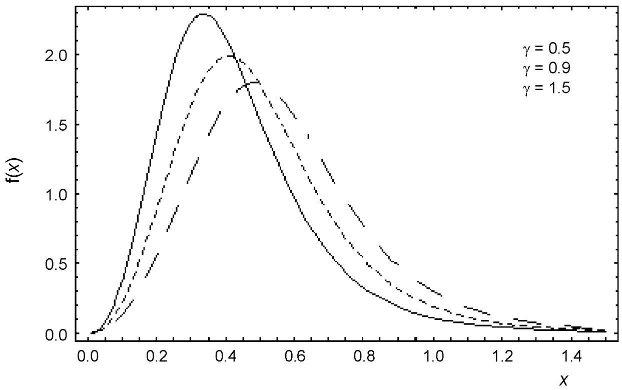

Fig. 2. Plot of pdf for different values of γ for $\mu = 0.01$, $\sigma = 1$, $\lambda = 0.5$, $m = k = 2$ and $p = 3$

IV. MOMENTS AND QUANTILES

The n-th moments, $n \ge 1$, can be computed to the *EB* distribution as follows:

$$E(X^n) = n\int_0^\infty x^{n-1}\bar{F}(x)\,dx =$$

$$= n\gamma \int_0^\infty \frac{\lambda^m \left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^{-m} x^{n-1}}{1-\bar{\gamma}\,\lambda^m \left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^{-m}}\, dx, \qquad (13)$$

in this calculation, we shall assume that $\mu = 0$ and $\sigma = 1$ without loss of generality.

Since $(1-x)^{-1} = \sum_{r=0}^{\infty} x^r$ and $(1-x)^r = \sum_{t=0}^{r} (-1)^t \binom{r}{t} x^t$, then

$$E(X^n) = n\gamma \int_0^\infty \lambda^m (\lambda + kx^p)^{-m} x^{n-1} \times$$

$$\times \sum_{r=0}^{\infty} (1-\gamma)^r \lambda^{mr} (\lambda + kx^p)^{-mr} = \qquad (14)$$

$$= n \sum_{r=0}^{\infty} \lambda^{m(r+1)} \sum_{t=0}^{r} (-1)^t \binom{r}{t} \gamma^{t+1} \int_0^\infty \frac{x^{n-1} dx}{(\lambda + kx^p)^{m(r+1)}}.$$

Thus from lemma 1, we get

$$E(X^n) = \left(\frac{n}{p}\right)\left(\frac{\lambda}{k}\right)^{\frac{n}{p}} \sum_{r=0}^{\infty}\sum_{t=0}^{r} (-1)^t \binom{r}{t} \gamma^{t+1} B\left(\frac{n}{p},\, m(r+1) - \frac{n}{p}\right). \qquad (15)$$

The q-th quantile of the *EB* distribution is given by

$$x_q = F^{-1}(q) = \left(\frac{\lambda}{k}\right)^{\frac{1}{p}} \sigma \times$$

$$\times \left(\left(1 - \frac{\gamma \left(q(\gamma-1)\lambda^m - \lambda^m + 2\left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m \right)}{q(\gamma-1)\lambda^m - \gamma\lambda^m + \left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m} \right)^{\frac{1}{m}} - 1 \right)^{\frac{1}{p}} + \mu, \qquad (16)$$

where $F^{-1}(.)$ is the inverse distribution function.

The median of the *EB* distribution, i.e. $x_{0.5}$, is given by

$$median(X) = \left(\frac{\lambda}{k}\right)^{\frac{1}{p}} \sigma \times$$

$$\times \left(\left(1 - \frac{\gamma \left(0.5(\gamma-1)\lambda^m - \lambda^m + 2\left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m \right)}{0.5(\gamma-1)\lambda^m - \gamma\lambda^m + \left(k\left(-\frac{\mu}{\sigma}\right)^p + \lambda\right)^m} \right)^{\frac{1}{m}} - 1 \right)^{\frac{1}{p}} + \mu \qquad (17)$$

V. FAILURE RATE FUNCTION

The concept of the FR in engineering (also known as the hazard rate in medical science and the force of mortality in actuarial science) is the basic staple of modern reliability theory.

The FR function of the $EB(\mu, \sigma, \gamma, \lambda, k, m, p)$ distribution is given by

$$h(x) = \frac{f(x)}{\bar{F}(x)} =$$

$$= \frac{kmp\left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^{m-1} \left(\frac{x-\mu}{\sigma}\right)^{p-1}}{\left((\gamma-1)\lambda^m + \left(k\left(\frac{x-\mu}{\sigma}\right)^p + \lambda\right)^m\right)\sigma}. \qquad (18)$$

The following remark presents a formal description of the possible shapes of the FR of the *EB* distribution in terms of the parameters $\mu, \sigma, \gamma, \lambda, k, m, p$.

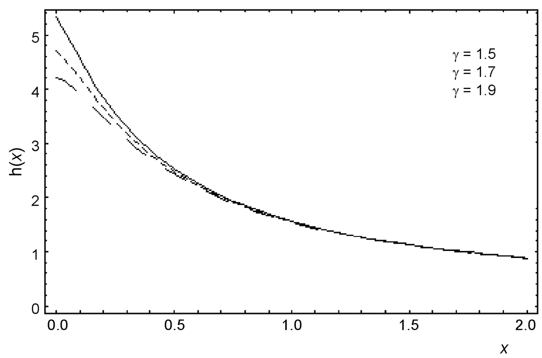

Fig. 3. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 0{,}5$, $m = k = 2$ and $p = 1$

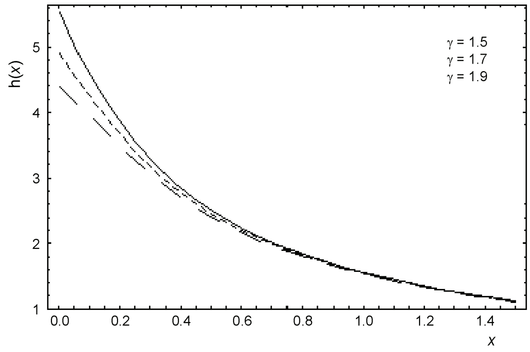

Fig. 4. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 0{,}5$, $m = k = 2$ and $p = 0.99$

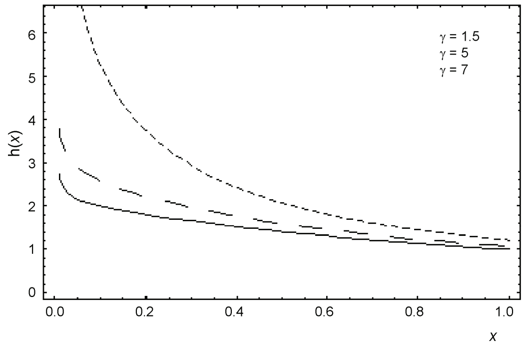

Fig. 5. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 2$, $m = k = 2$ and $p = 0.77$

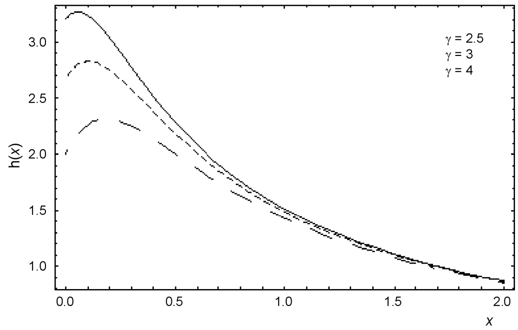

Fig. 6. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 0.5$, $m = k = 2$ and $p = 1$

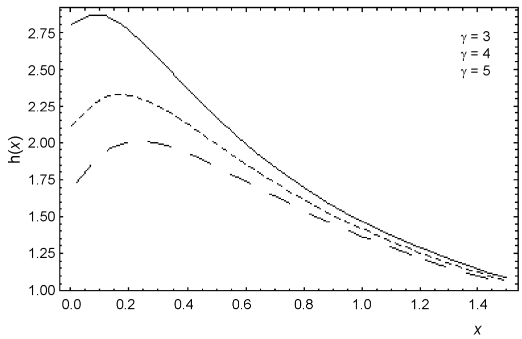

Fig. 7. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 0.5$, $m = k = 2$ and $p = 0.99$

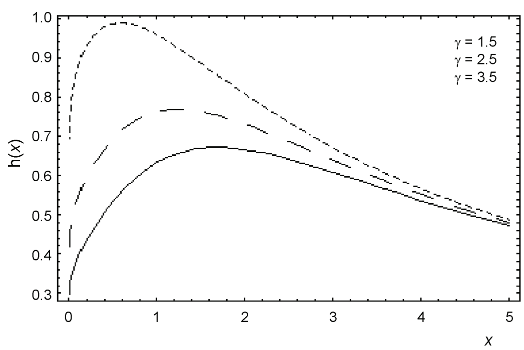

Fig. 8. Plot of FR for different values of γ for $\mu = 0$, $\sigma = 1$, $\lambda = 2$, $m = 3$, $k = 1$ and $p = 1.1$

Remark

If $X \sim EB\left(\mu,\sigma,\gamma,\lambda,k,m,p\right)$, then X has

(i) inecreasing-decreasing FR provided
 (a) $p \le 1, \gamma > 2,$ or
 (b) $p > 1$.

(ii) decreasing FR provided
 (a) $p \le 1, \gamma < 2$, or
 (b) $p \le 0.77$.

VI. STOCHASTIC ORDERINGS

Stochastic ordering of positive continuous random variables is an important tool to judge the comparative behaviour. We shall recall some basic definitions.

Random variable X is said to be smaller than random variable Y in the

(i) stochastic order (denoted by $X \le_{st} Y$) if $\overline{F}_X\left(x\right) \le \overline{F}_Y\left(x\right)$ for all x;
(ii) hazard rate order (denoted by $X \le_{hr} Y$) if $h_X(x) \ge h_Y(x)$ for all x;
(iii) likelihood ratio order (denoted by $X \le_{lr} Y$) if $f_X(x)/f_Y(x)$ decreases in x.

The following implications, see Ross [10] chapter 9, are well known:

$$X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{st} Y. \tag{19}$$

The *EB* distributions are ordered with respect to the strongest "likelihood ratio" ordering as shown in the following theorem.

Theorem 3

Let $X \sim EB\,(\mu, \sigma, \gamma_1, \lambda, k, m, p)$ and $Y \sim EB\,(\mu, \sigma, \gamma_2, \lambda, k, m, p)$. If $\gamma_1 < \gamma_2$, then

$$X \le_{lr} Y\left(X \le_{hr} Y, X \le_{st} Y\right). \tag{20}$$

Proof

First, note that

$$\frac{f_X(x)}{f_Y(x)} = \frac{\gamma_1}{\gamma_2}\left[\frac{\left(\left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m - \overline{\gamma}_2\lambda^m\right)}{\left(\left(\lambda + k\left(\frac{x-\mu}{\sigma}\right)^p\right)^m - \overline{\gamma}_1\lambda^m\right)}\right]^2, \quad x > 0.$$

Since, for $\gamma_1 < \gamma_2$,

$$\frac{d}{dx}\frac{f_X(x)}{f_Y(x)} = 2kmp\lambda^m \frac{\gamma_1}{\gamma_2}(\gamma_1-\gamma_2) \times$$

$$\times \frac{\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^{m-1}\left(\frac{x-\mu}{\sigma}\right)^{p-1}\left(\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^m-\overline{\gamma}_2\lambda^m\right)}{\sigma\left(\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^m-\overline{\gamma}_1\lambda^m\right)^3}$$

$$< 0, \qquad (22)$$

then $f_X(x)/f_Y(x)$ is decreasing in x. That is $X \leq_{lr} Y$. The remaining statements follow from the implications (19).

VII. LIMITING DISTRIBUTIONS OF SAMPLE EXTREMES

Let $X_1, X_2, \ldots, X_n$ be a random sample of size *n* from an absolutely continuous distribution with pdf $f(x)$ and cdf $F(x) = 1-\overline{F}(x)$. Limiting distribution of sample maxima $X_{n:n} = \max(X_1, X_2, \ldots, X_n)$ is a long stand area in applications of probability and statistics (see, e.g. Kotz and Nadarajah [3]). First, we recall the following asymptotical results for $X_{n:n}$ (see, e. g., Arnold et al. [1], pp. 210-214).

(i) For the maximal order statistic $X_{n:n}$, we have

$$\lim_{n\to\infty} P\{X_{n:n} \leq a_n + b_n t\} = \exp(-e^{-t}), \qquad (23)$$

$$-\infty < t < \infty,$$

(of the extreme value type) where $a_n = F^{-1}(1-1/n)$ and $b_n = 1/nf(a_n)$ if

$$\lim_{x\to\infty}\frac{d}{dx}\left(\frac{1}{h(x)}\right) = 0. \qquad (24)$$

The following theorem derives the limiting distributions of the largest order statistics from the *EB* distribution.

Theorem 5.

Let $X_{n:n}$ be the largest order statistics from $EB(\mu, \sigma, \gamma_1, \lambda, k, m, p)$ distribution. Then

$$\lim_{n\to\infty} P\{X_{n:n} \leq a_n + b_n t\} = \exp\left(-e^{-t}\right), \quad -\infty < t < \infty, \qquad (25)$$

where $a_n = F^{-1}(1-1/n)$ and $b_n = 1/(n f a_n)$ and $f(.), F^{-1}(.)$, respectively, are given by (11) and (16).

Proof.

For *EB* distribution, we have

$$\lim_{x\to\infty}\frac{d}{dx}\left(\frac{1}{h(x)}\right) =$$

$$= \lim_{x\to\infty}\frac{\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^{-m}\left(\frac{x-\mu}{\sigma}\right)^{-p}}{kmp} \times \qquad (26)$$

$$\times\left((1-p)\lambda\left(\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^m-\overline{\gamma}\lambda^m\right)+\right.$$

$$\left.+\; k\left((mp-1)\overline{\gamma}\lambda^m+\left(k\left(\frac{x-\mu}{\sigma}\right)^p+\lambda\right)^m\right)\left(\frac{x-\mu}{\sigma}\right)^p\right) = 0.$$

Hence, the statement follows from (23) and (24).

VIII. CONCLUSION

The new extended Burr XII distribution with the reliability function is obtained. The proposed extended Burr XII distribution has a unimodal probability density function with no or one critical point. Such characteristics are useful for modeling continuous data from life-testing experiments. Some reliability properties are obtained. Moreover, closed forms for *n*-th moments are derived. The extended Burr XII distribution is ordered with respect to the likelihood ratio stochastic ordering. The limiting distribution of the largest order statistics of a random sample from the proposed distribution has an extreme-value distribution.

The width and amplitude of the distribution profile have been controlled by the seven parameters (locations, and scales), confirmed by Figures (1-2).

Figures (1-2) show that the increasing of γ decreasing the amplitude and increasing the width while increasing of *p* increasing the amplitude and decreasing the width.

On the other hand, Figures (3-8) show the variations of FR for different values of γ and *p*, it is clear that for (a) $p \leq 1,\ \gamma > 2,$ or (b) $p > 1$, inecreasing-decreasing FR is obtained while for $p \leq 1, \gamma < 2,$ or (b) $p \leq 0.77$ decreasing FR is obtained.

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